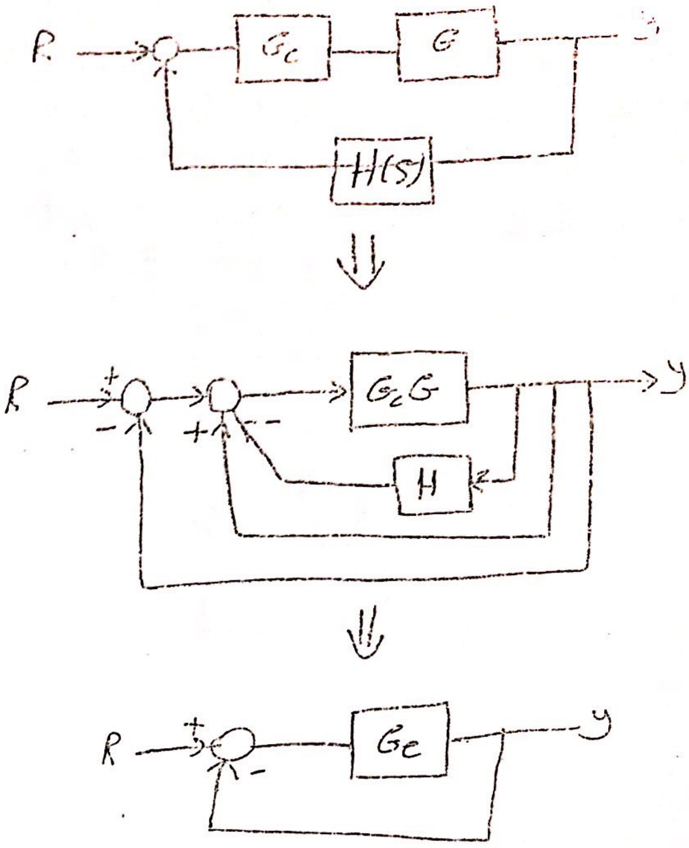
Tracking error with non-unity feedback :- CE 306 Final

ما بعد المحاضرة (8) الأخيرة

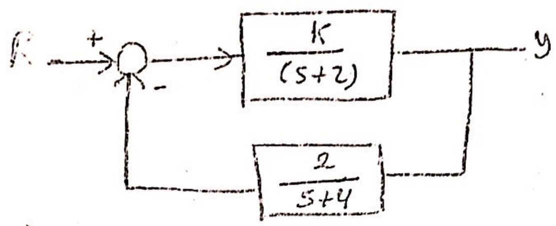
$$* \; G_e = \frac{G G_c}{1 + G G_c (H - 1)}$$



* Example@2

① - find $e_{ss}$ for unit step,

② - What Value of $K$, $e_{ss} = 0$ ?



① $e_{ss} = \lim_{s \to 0} s [1 - T(s)] \frac{1}{s}$

$$\because T(s) = \frac{\frac{K}{s+2}}{1 + \frac{2K}{(s+2)(s+4)}}$$

$$= \frac{K(s+4)}{(s+2)(s+4) + 2K}$$

$$\rightarrow e_{ss} = \lim_{s \to 0} 1 - \frac{K(s+4)}{(s+2)(s+4) + 2K}$$

$$= \lim_{s \to 0} \frac{(s+2)(s+4) + 2K - K(s+4)}{(s+2)(s+4) + 2K}$$

$$= \frac{(2)(4) + 2K - 4K}{(2)(4) + 2K} = \frac{8 - 2K}{8 + 2K}$$

② for $e_{ss} = 0$ :-

$$8 - 2K = 0$$

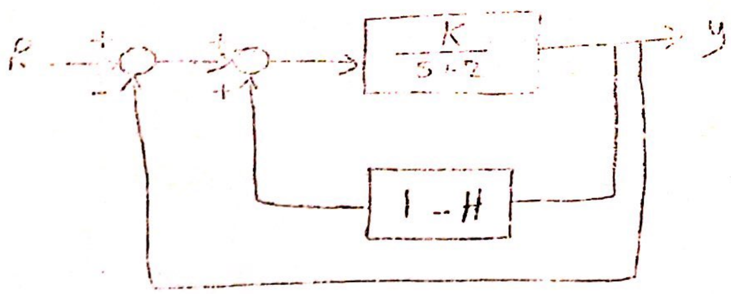
$$\rightarrow K = \frac{8}{2} = 4$$

EE 362

Method (2): for Solution.

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{1 + \frac{K(s+4)}{[(s+4)-K](s+2)}} \cdot \frac{1}{s}$$

$$= \frac{2(4-K)}{8+2K} = \frac{8-2K}{8+2K}$$

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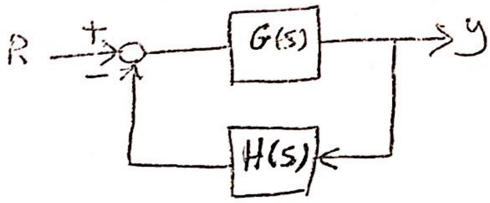
# The root locus:




- The ch.eq is:

$$1 + G(s)H(s) = 0$$

let we represent $G(s)H(s)$ as:

$$G(s)H(s) = KL(s) \quad , K \geq 0$$

Multiplcation

$$\rightarrow L(s) = \frac{\prod_{i=1}^{m} (s - z_i)}{\prod_{i=1}^{n} (s - P_i)} \quad , \quad m < n \text{ for proper T.F.}$$

We try to find the root locus of $1 + KL(s) = 0$ of C.L.S. as K changes from 0 to infinity.

If "s" is a root, then $KL(s) = -1$

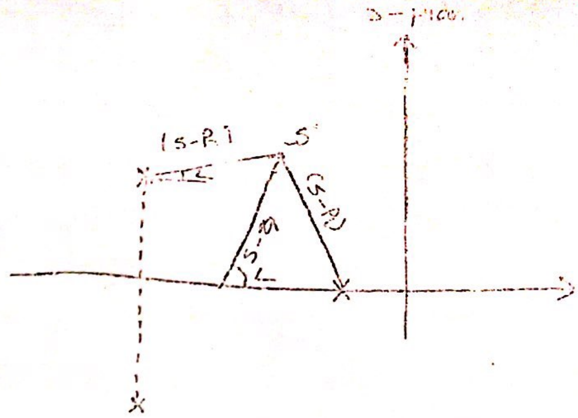
* Magnitude criterion:

$$|KL(s)| = 1$$

* phase (angle) criterion:

$$\angle L(s) = (2\bar{K}+1)180^\circ \quad , \bar{K} = 0, \pm 1, \pm 2, \ldots \infty$$

$$\text{or } \angle L(s) = \pm 180^\circ$$




* Magnitude criterion:-

$$K \frac{\prod_{i=1}^{m} |s-z_i|}{\prod_{i=1}^{n} |s-P_i|} = 1 \quad ; \quad K \cdot \frac{\text{product of distances from zeros}}{\text{product of distances from poles}} = 1$$

* phase criterion:-

Sum of angles from zeros - sum of angles from poles

$$= (2K+1)\,180^\circ \quad , K = 0, \pm 1, \pm 2, --$$

$$m \leq n$$

$$1 + K \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-P_i)} = 0$$

$$\therefore \prod_{i=1}^{n}(s-P_i) + K \prod_{i=1}^{m}(s-z_i) = 0 \quad \text{——} \ (1)$$

- n branches of the root locus, They start from open-loop poles at $K = 0$.

→ by dividing (1) by K :-

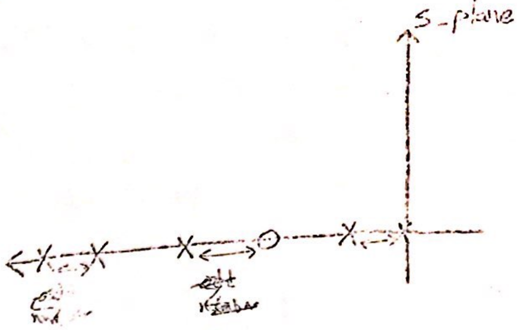
$$\lim_{K \to \infty} \left( \frac{1}{K} \prod_{i=1}^{n}(s-P_i) + \prod_{i=1}^{m}(s+z_i) \right) = 0$$

- m branches will approch the open-loop zeros $z_1, z_2, --, z_m$
- The remaining $(n-m)$ branches approch $\infty$ at asymptotes.

To draw the root locus, we follow these steps:-

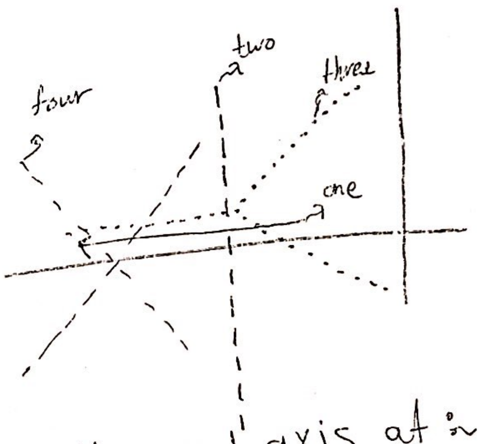
1] Root locus on the real axis :-

Any segment of the real axis to the left of an odd number of poles and zeros counted together, lies on the root locus




2] Roots which approch ∞ do so along asymptots :-

| | | |
|---|---|---|
| $n-m=0$ | no asymptots | |
| $n-m=1$ | one // | |
| $n-m=2$ | two // | $(\pm 90)$ |
| $n-m=3$ | Three // | $(60^\circ, 180^\circ, -60^\circ)$ |
| $n-m=4$ | foure // | $(45^\circ, 135^\circ, -135^\circ, -45^\circ)$ |

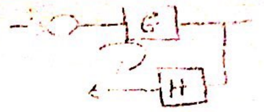



The asymptotes intersect the real axis at :-

$$\sigma_a = \frac{\Sigma \text{ Poles of } L(s) - \Sigma \text{ zeros of } L(s)}{n-m}$$

Example (1):

Draw the root locus for: $KL(s) = \frac{K}{s(s+2)}$

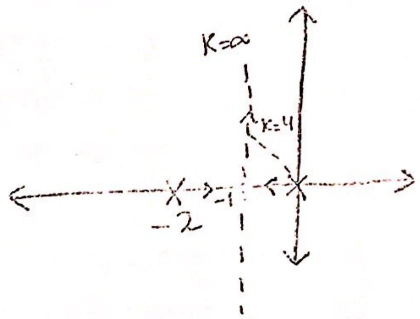



open loop gain $= GH = KL(s)$

Sol:-

- no of asymptotes = 2.

$$\sigma_a = \frac{-2-0-(0)}{2} = -1$$



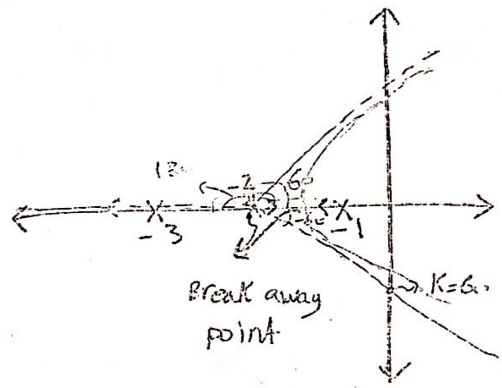

#Example(2):

$$KL(s) = \frac{K}{(s+1)(s+2)(s+3)}$$ , sketch the root locus?

Sol:-

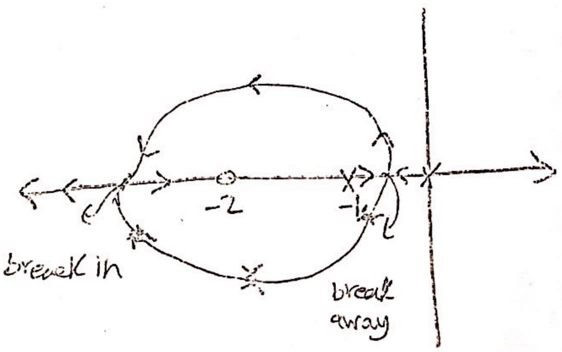
$N - M = 3$ , we have 3 asymptotos (60, -60, 180)

$$\sigma_a = \frac{\Sigma P - \Sigma Z}{n-m} = \frac{-6}{3} = -2$$



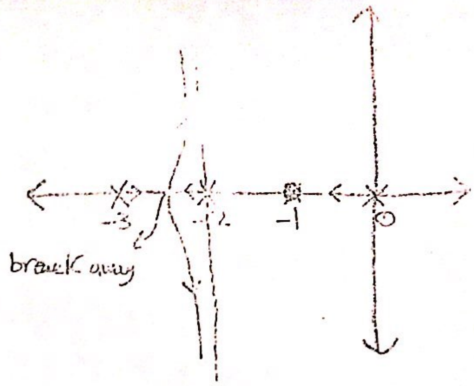

#Example(3):

$$KL(s) = \frac{K(s+2)}{s(s+1)}$$



# Example (4) :-

$$KL(s) = \frac{K(s+1)}{s(s+2)(s+3)}$$




Sol:-

$$\sigma_a = \frac{\Sigma P - \Sigma Z}{n-m} = \frac{-5-(-1)}{3-1}$$

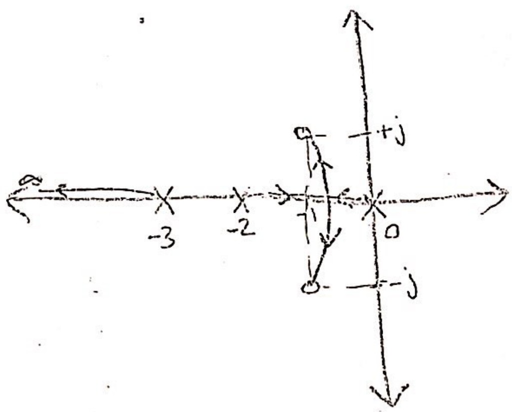
$$= -2$$

# Example (5) :-

$$KL(s) = \frac{K(s^2+2s+2)}{s(s+2)(s+3)}$$

$$s_{1,2} = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm j$$




dynamic system → has storge element

Resistance is memory less function (is not dynamic)

## Intersection with Imagainary Axis:

Use Routh-Hurwitz criterion to find the point of intersection

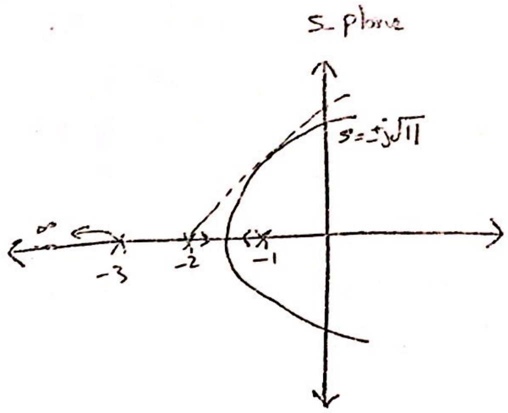
→ Ex (6):

$$KL(s) = \frac{K}{(s+1)(s+2)(s+3)}$$

$$\therefore s^3 + 6s^2 + 11s + 6 + K = 0$$

| | | |
|---|---|---|
| $s^3$ | 1 | 11 |
| $s^2$ | 6 | $6+K$ |
| $s$ | $\frac{1}{6}(60-K)$ | |
| $1$ | $6+K$ | |

→ at $K = 60$, we have roots on the Imaging axis



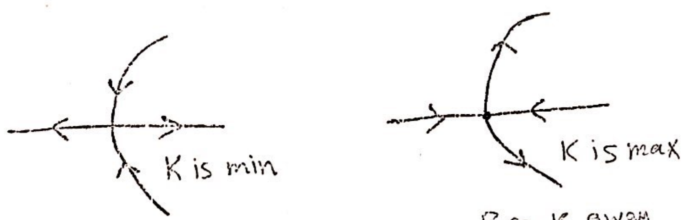

* from auxiallary equation:

$$6s^2 + 66 = 0$$

$$s^2 + 11 = 0$$

$$s = \pm j\sqrt{11}$$

# Break in and Break away points :-




Break away

\# Break in

$$K = \frac{-1}{L(s)} , \quad \frac{d}{ds}\left[\frac{-1}{L(s)}\right] = 0$$

→ Ex (7):-

$$KL(s) = \frac{K}{(s+1)(s+2)(s+3)}$$

$$\frac{d}{ds}\left[(s+1)(s+2)(s+3)\right] = 0$$

$$\frac{d}{ds}\left[s^3 + 6s^2 + 11s + 6\right] = 0$$

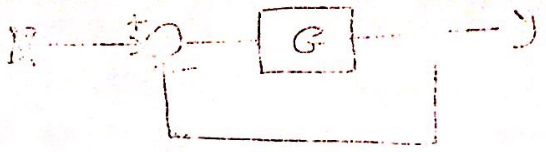
$$3s^2 + 12s + 11 = 0$$

→ Roots are $-1.423$ and $-2.577$

this is true value, because is between $-1$ & $-2$.

Ex(8):

$$G(s) = \frac{1}{s^2 + s + K}$$




Sketch the root locus for $K = 0 \to \infty$?

→ Sol :-

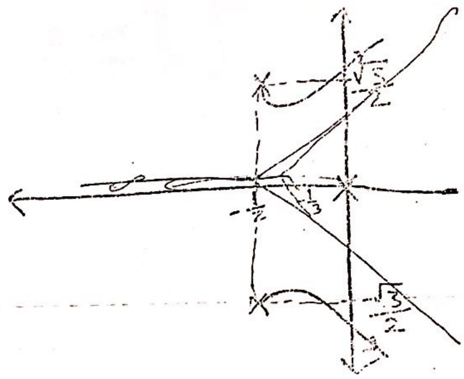
$$T(s) = \frac{G}{1+GH} = \frac{\frac{s}{s^3+s^2+K}}{1+\frac{s}{s^3+s^2+K}} = \frac{s}{s^3+s^2+s+K}$$

$$\therefore\ s^3+s^2+s+K=0$$

$$(s^3+s^2+s)+K=0$$

$$1+\frac{K}{s^3+s^2+s}=0$$

$$1+\frac{K}{s(s^2+s+1)}$$




$n=3$ , $m=0$

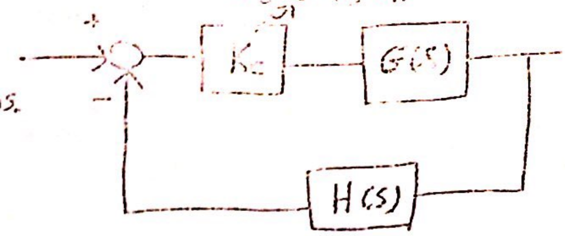
$n-m=3$ asymptotes

$$\sigma_a = \frac{\Sigma P - \Sigma Z}{n-m} = -\frac{1}{3}$$

# Design Using the root locus:

هو اختيار قيمة الـ K بحيث
تعطينا شروط التصميم

Given $G(s)$ and $H(s)$ design $K_c$ to meet the design specifications.




※ Design specifications:

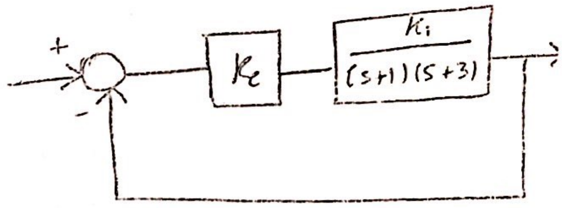
1- Steady_state tracking accuracy, $e_{ss}$, $e(\infty)$ OR $K_p$, $K_v$, $K_a$.

2- Transient response: - overshoot
- $t_r$, $t_p$, $t_s$

Translate the transient response into requirements on $(\xi, \omega_n)$ of a pair of dominant complex poles.

⇒ Example(1):

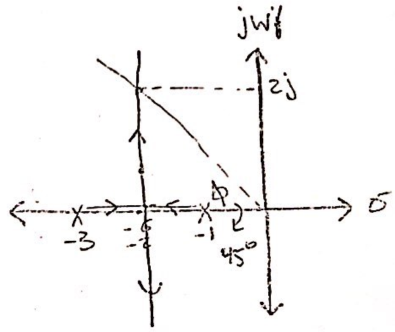
Design $K_c$ such that $P.O \leq 5\%$




We know:

| over shoot | $\xi$ |
|---|---|
| 5% | $\frac{1}{\sqrt{2}}$ |
| 16.3% | 0.5 |

Then, take $\xi = \frac{1}{\sqrt{2}} \Rightarrow \phi = \cos^{-1}(\frac{1}{\sqrt{2}}) = 45^\circ$




∵ from S_plane → $s = -2 \pm j2$

To calculate $K_c$, we use Magnitude criterion:

$|KL(s)| = 1$ , where $K = K_c K_1$.

$$\left|\frac{K}{(s+1)(s+3)}\right| = 1$$

$$K = |(s+1)(s+3)|_{s=-2+j2}$$

$$= |(-2+2j+1)(-2+j2+3)|$$

$$= |(-1+j2)(1+j2)|$$

$$K = 5$$

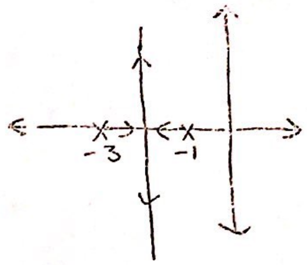
$$K_c = \frac{5}{K_1}$$ ?

# Matlab:

```
>> h = (1, [1 4 3])
>> rlocus (h)
>> rlocfind (h)
```

Example (2): for the same system, Design $K_c$ such that the $e_{ss}$ to a Unit step input $\leq 5\%$ ?

Solution




$$KL(s) = \frac{K}{(s+1)(s+3)}$$

It is stable for $0 \leq K \leq \infty$

$$K_p = G(0) = \frac{K}{3}$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\frac{K}{3}} \leq 0.05$$

$$\frac{3}{3+K} \leq 0.05 \Rightarrow 3+K \leq \frac{1}{0.05} \times 3$$

$$K \leq 60-3$$

$$K_? \leq 57$$

$$\therefore K_p = \frac{57}{3} = 19$$

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⟹ Example (3): for the same system, design $K_c$ such that

1) overshoot $\leq 5\%$ $[K \leq 5]$

2) $e(\infty)$ for step $\leq 5\%$ $[K \leq 57]$

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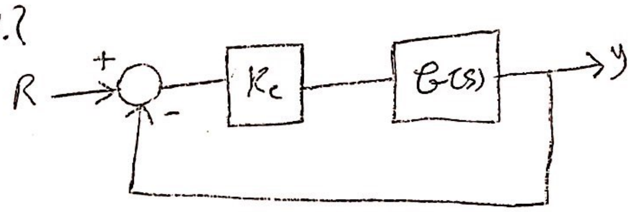
⟹ Example (4): Design $K_c$ s.t overshoot $\leq 20\%$ ?




take $\xi = 0.5 \rightarrow \phi = 60°$

$\sigma \simeq 0.4 \rightarrow$ $\omega_d = \frac{0.4}{0.5}\sqrt{1-(0.5)^2} = 0.6928$

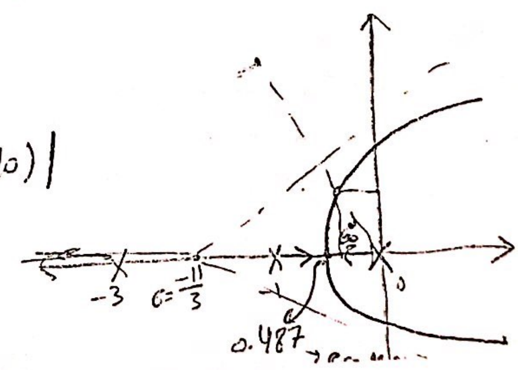
$$G(s) = \frac{10}{s(s+1)(s+10)}$$

$\therefore s = -0.4 + j0.6928$

$$10 K_c = |s(s+1)(s+10)|$$

$$= |(-0.4+j0.6928)(-0.4+j0.6928+1)(-0.4+j0.6928+10)|$$

$$K_c = 0.7$$



PID Controller (3-Term Controller) ~

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s$$

* Analytic design (Use a plant model $G(s)$) ~

$$G_c(s) = \frac{K_D s^2 + K_p s + K_I}{s} = \frac{K_D (s^2 + \frac{K_p}{K_D} s + \frac{K_I}{K_D})}{s}$$

Choose the gain such that the numerator polynominal has two real roots.

$$G_c(s) = \frac{K(s+z_1)(s+z_2)}{s} \quad ; \text{ where: } K = K_D$$

$$K(z_1 + z_2) = K_p$$

$$K z_1 z_2 = K_I$$

$$G_c(s) = \underbrace{K(s+z_1)}_{PD} \underbrace{\left(\frac{s+z_2}{s}\right)}_{PI}$$

Design $K$ & $z_1$ to meet the transient response specs. Then choose $z_2$ small to preserve the transient response.

⇒ Example ~

$$G(s) = \frac{0.5}{s(s+2.5)} \quad , H(s) = 1$$

Specs:
1) overshoot $\leq 20\%$
2) $T_p \leq 0.5$ sec
3) $K_v \geq 100$

Design the PID controller to meet the above specs?

Example :

$G(s) = \dfrac{0.5}{s(s+2.5)}$ , Design a controller , such that :

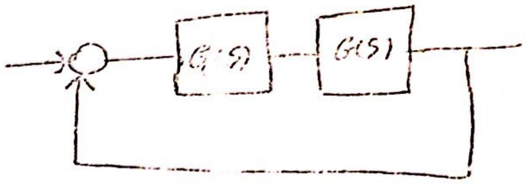
$H(s) = 1$

Specs :- 1) overshoot $\leq 20\%$

2) $T_p \leq 0.5$ sec

3) $K_v \geq 100$

Solution :- *) first Using a controller with gain $K_c$ :




let $\xi = 0.5$

$\phi = 60^\circ$

$s = -1.25 + j1.25\sqrt{3}$

$T_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{1.25\sqrt{3}} > 0.5$

# Conclusion :- $K_c$ is not going to work.

*) second : Design the PD compensator to shift the root locus to the left :

$G_c(s) = K(s + z_1)$

→ to find the designed roots :

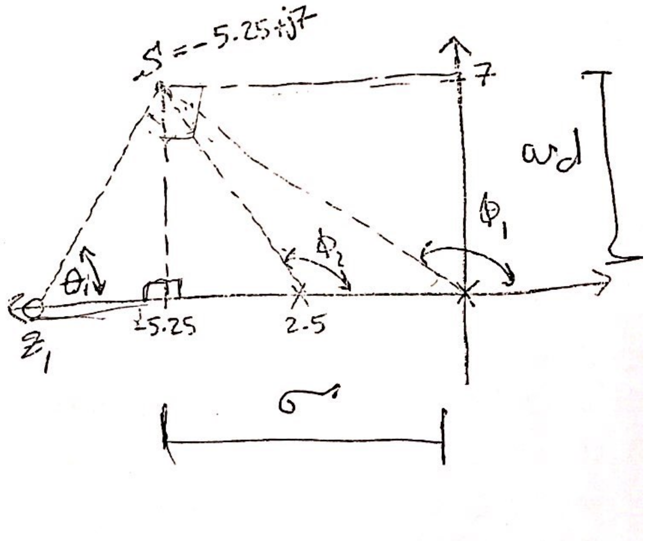
$T_p = \dfrac{\pi}{\omega_d} \leq 0.5 \longrightarrow \omega_d \geq \dfrac{\pi}{0.5} = 6.28$

Take $\omega_d = 7$

Take $\xi = 0.6$

$\therefore \omega_n = \dfrac{\omega_d}{\sqrt{1-\xi^2}} = 8.75$

$\therefore \sigma = \xi \omega_n = 5.25$




⇒ Angle criterion :

$\theta_1 - (\phi_1 + \phi_2) = \pm 180$

$\theta_1 - \left(\left(180 - \tan^{-1}\frac{7}{5.25}\right) + \left(180 - \tan^{-1}\frac{7}{2.75}\right)\right) = 180$

$\therefore \theta_1 = 58.3^\circ$

$\because \tan(58.3) = \frac{7}{z_1 - 5.25}$

$1.619 = \frac{7}{z_1 - 5.25} \rightarrow z_1 = 9.57 \simeq 9.5$

$G_c(s) = K(s+9.6)$

$$GG_c = \frac{0.5K(s+9.6)}{s(s+2.5)}$$

$$\because \left| \frac{0.5K(s+9.6)}{s(s+2.5)} \right|_{s=-5.25+j7} = 1 \longrightarrow K \simeq 16$$

# Matlab check:

$T_p \simeq 0.3\ sec \Rightarrow$ overshoot $\simeq 17.5$ (✓) $\Rightarrow K_v < 100$ (X)

So, PD controller in not enough.

*) Three: we added a PI term: $\frac{s+z_2}{s}$

Such that $|z_2| = 0.1 \times Re(s)$

$z_2 = 0.1 \times 5.25 \simeq 0.5$

$$G_c = \frac{16(s+9.6)(s+0.5)}{s}$$

# Matlab check:

$T_p \simeq 0.3 \rightarrow$ overshoot $\simeq 21\%$ (X) $\rightarrow e_{ss} = 0$

$\Rightarrow$ Take $z_2 = -0.4$

$\rightarrow T_p \simeq 0.3\ sec \rightarrow$ Overshoot $\simeq 20\% \rightarrow e_{ss} = 0$ (✓)

# PID tuning :-

No Analytical model of G(s)

| PID gain | % OS | $T_s$ | $e_{ss}$ |
|---|---|---|---|
| $K_p$ | ↑ | minimal effect | ↓ |
| $K_D$ | ↓ | ↓ | No effect |
| $K_I$ | ↑ | ↑ | zero $e_{ss}$ |

*) one possible tuning procedue :-

1- Tune $K_p$ to get a stable response, possibly with overshoot large than desired.

2- Tune $K_D$ to meet the transient response specs.

3- Tune $K_I$ (small enough) to have a little effect on transient response

# Frequency Response

- The steady-state response of the system to a sine-input.
- The output can be obtained by subsituting $s = jw$ in the closed-loop system relationship, $Y(s) = T(s)R(s)$

$$Y(jw) = T(jw)\,R(jw) = \frac{G(jw)}{1 + G(jw)H(jw)}\,R(jw)$$

- can be used to analyze high order system.

## Frequency response plots:

### 1] Polar Plots:

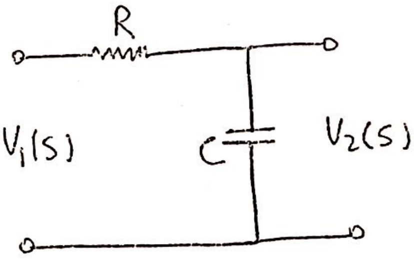
$$G(jw) = G(s)\Big|_{s=jw} = R(w) + jX(w) \;; \text{ where:}$$

$$R(w) = Re[G(jw)]$$
$$X(w) = Im[G(jw)]$$

In polar form:

$$G(jw) = |G(jw)|\, e^{j\phi(w)}$$
$$= |G(jw)| \angle \phi(w) \;; \text{ where: } \phi(w) = \tan^{-1}\frac{X(w)}{R(w)}$$
$$|G(w)|^2 = [R(w)]^2 + [X(w)]^2$$

Example (1):
Draw the polar plot for $G(s)$ of the circuit shown:




$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{RCs + 1}$$

$$G(jw) = \frac{1}{RC\,jw + 1} \;; \; RC = \frac{1}{w_1}$$

$$= \frac{1}{j\frac{w}{w_1} + 1}$$

ن المقام

$$|G(w)| = \frac{1}{\sqrt{\left(\frac{w}{w_1}\right)^2 + 1}} \;, \quad \phi = -\tan^{-1}\left(\frac{w}{w_1}\right)$$

$$G(j\omega) = R(\omega) + jX(\omega)$$

$$= \frac{1 - j\left(\frac{\omega}{\omega_1}\right)}{\left(\frac{\omega}{\omega_1}\right)^2 + 1} = \underbrace{\frac{1}{\left(\frac{\omega}{\omega_1}\right)^2 + 1}}_{R(\omega)} - j\underbrace{\frac{\left(\frac{\omega}{\omega_1}\right)}{\left(\frac{\omega}{\omega_1}\right)^2 + 1}}_{X(\omega)}$$

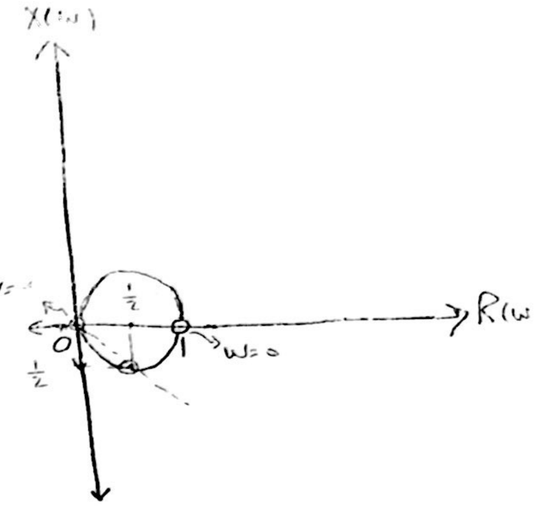
Now:-

at $\omega = 0 \Rightarrow R(\omega) = 1$ , $X(\omega) = 0$

at $\omega = \infty \Rightarrow R(\omega) = 0$ , $X(\omega) = 0$

$$\frac{\frac{\omega}{\omega_1}}{\left(\frac{\omega}{\omega_1}\right)^2} = \frac{1}{\omega_1} = 0$$

at $\omega = \omega_1 \Rightarrow \frac{1}{\sqrt{2}} \angle -45$



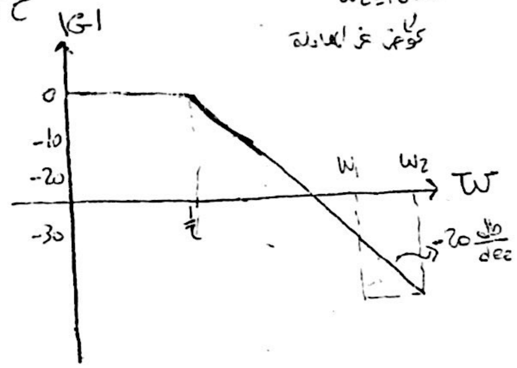

# Bode-plot :-

The introduction of logarithmic plot's, often called "Bode-plot's" simplifies determination of freq response :

$$\text{logarithmic gain} = 20 \log_{10} |G(j\omega)| \ db$$

Ex(1) :- $G(j\omega) = \frac{1}{j\omega(RC) + 1} = \frac{1}{j\omega\tau + 1}$ , $\tau = RC$

let : $\omega_1$, $\omega_2 = 10\omega_1$
كوكبر غ العاشرة




$$20 \log |G| = 20 \log \left(\frac{1}{1 + (\omega\tau)^2}\right)^{\frac{1}{2}}$$

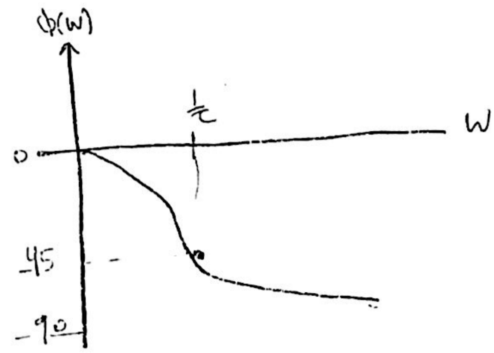
$$= -10 \log \left(1 + (\omega\tau)^2\right)$$

→ when $\omega \ll \frac{1}{\tau}$ : $20 \log |G| = 0$ , $\phi(\omega) = 0$

→ When $\omega \gg \frac{1}{\tau}$ : $-20 \log(\omega\tau)$

→ when $\omega = \frac{1}{\tau}$ : $-10 \log(2) = -3db$ , $\phi(\omega) = -45°$

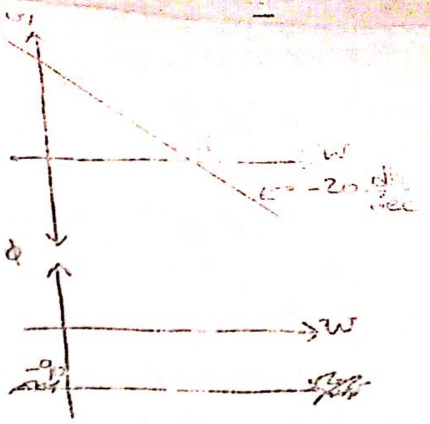
$\because \phi(\omega) = -\tan^{-1}(\omega\tau)$



$$G(s) = \frac{K}{s} \quad , \quad G(j\omega) = \frac{K}{j\omega}$$

$$|G| = \frac{K}{\omega} \quad , \quad \angle G = -90°$$

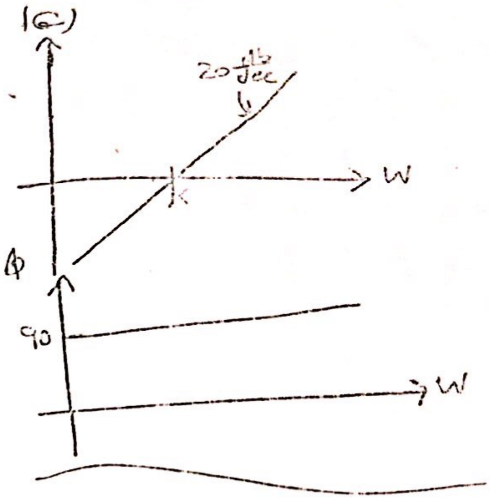
$$20\log|G| = 20\log\frac{K}{\omega}$$



Ex(3):

$G(s) = Ks$ , $G(j\omega) = jK\omega$

similar to integrator, we can obtain.



⇒ one advantage of B.P is that it simplifies gain multipication:

$$G = G_1 G_2 G_3$$

$$|G| = |G_1| * |G_2| * |G_3|$$

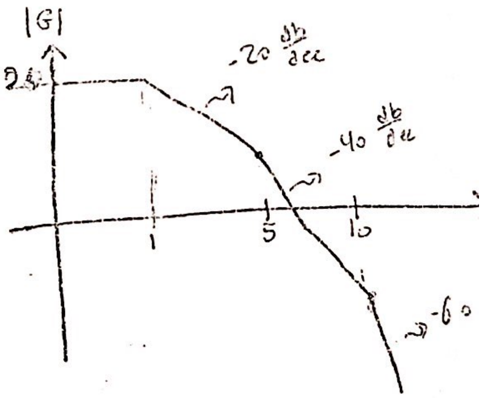
$$\angle G = \angle G_1 + \angle G_2 + \angle G_3$$

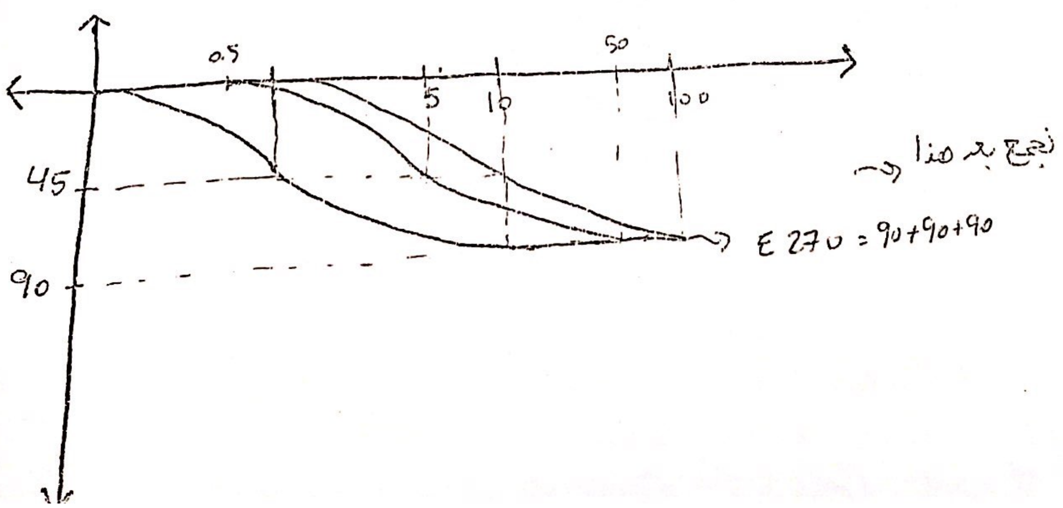
$$20\log|G| = 20\log|G_1| + 20\log|G_2| + 20\log|G_3|$$

Ex(4):

$$G(s) = \frac{1000}{\underbrace{(s+1)(s+5)(s+10)}_{\text{freq corner}}} = \frac{20}{(s+1)(\frac{1}{5}s+1)(\frac{1}{10}s+1)}$$

$$20\log 20 = 20\log 10 + 20\log 2 = 26\,db$$



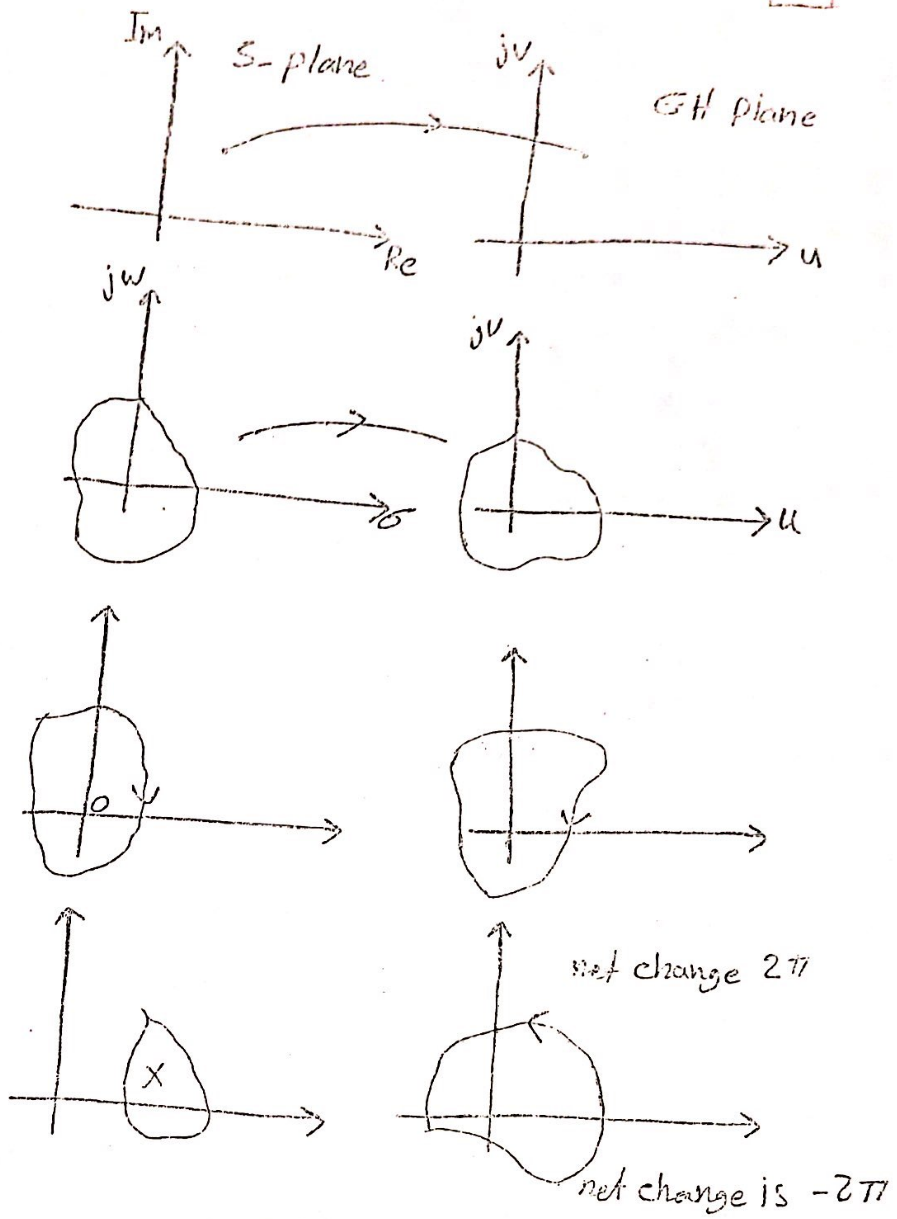
Second order term :-

$$G(S) = \frac{W_n^2}{S^2 + 2\xi W_n S + W_n^2}$$

... in the frequency response

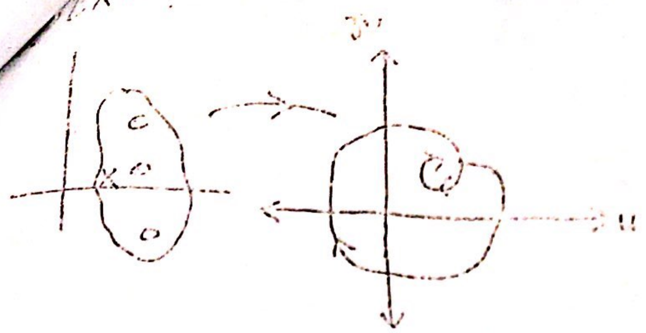
$$T(s) = \frac{G}{1+GH}$$

$$GH = K\frac{\prod^{m}(s+z_i)}{\prod^{n}(s+p_i)}$$


Net change in $\angle GH = 2\pi(Z-P)$

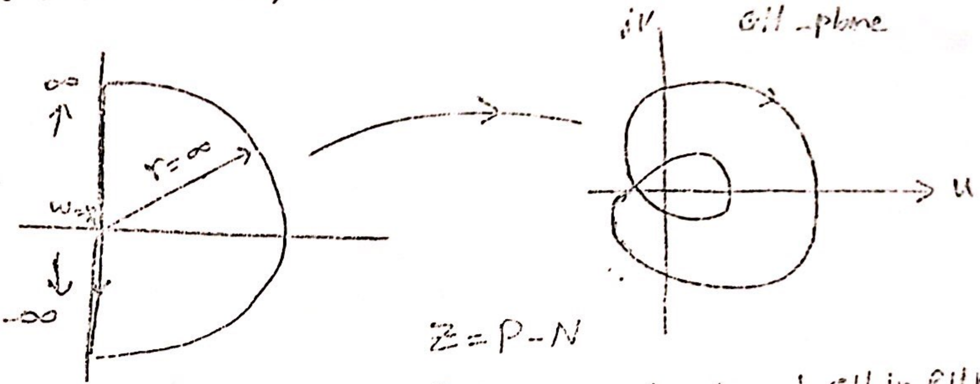
Z is number of zeros
P is number of poles



# The nuquist criterion :-

$1 + GH = 0$

Draw $GH(s)$ with contour covers RHP of s-plane :-




$Z = P - N$

$P \triangleq$ number of poles of GH in RHP

$N \triangleq$ number of counter-clockwise encirclement of $-1+j0$ in GH-plane

# Relative stability

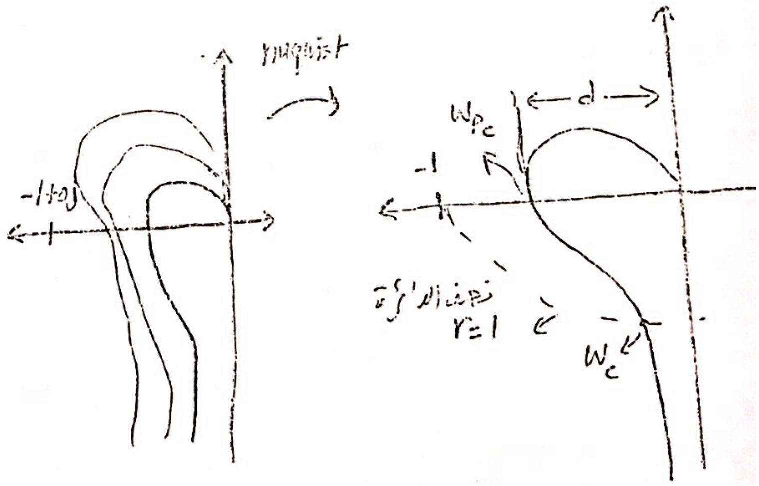
The idea which will be used with bode plot, let GH has ~~two~~ no poles in the RHP, $P = 0$.

Gain margin $= 20 \log \frac{1}{d} = GM$

phase Margin $= \phi_M$

$W_{P_c}$ = phase cross-over frequency

$W_c$ = cross-over frequency

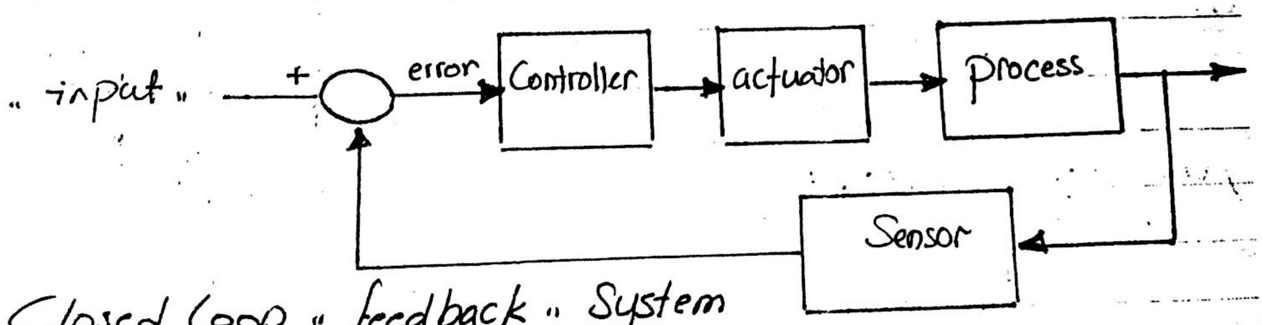


# EE362

Control systems — Lecture #

* Definition :— Consists of subsystems and process(plant) assembled for the purpose of obtaining desired output with desired performance.
Control System is One where the Output of the System is Controlled to be at some specific values (regulation), or Changes in some prescriped way (tracking) as determined by the System.

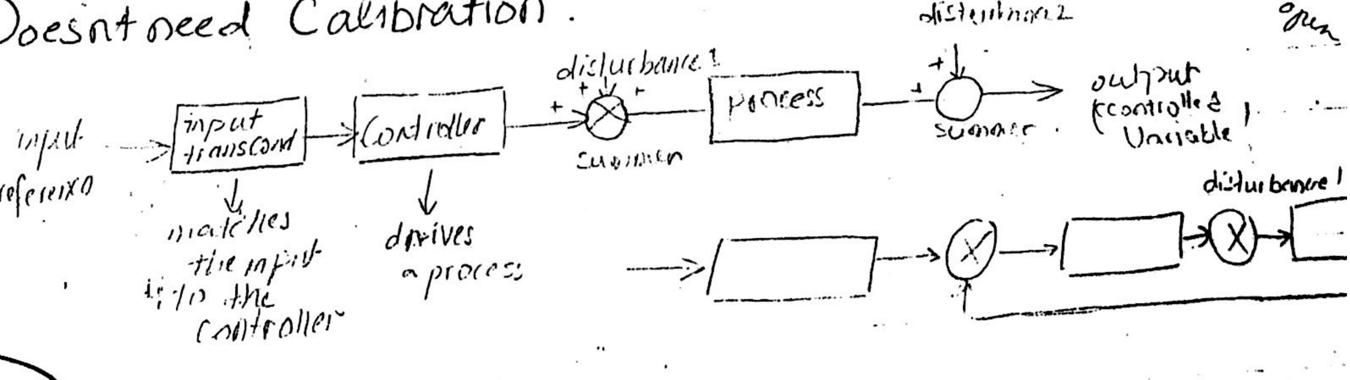
* Components of the Control system :—
1. process (plant):
2. Actuator.
3. Sensor (measurement element)
4. Controller.




Closed Loop " feedback " System

* Advantages :—
- Accuracy
- Disturbance Rejection
- Stablize non-stable system
- Doesnt need Calibration.

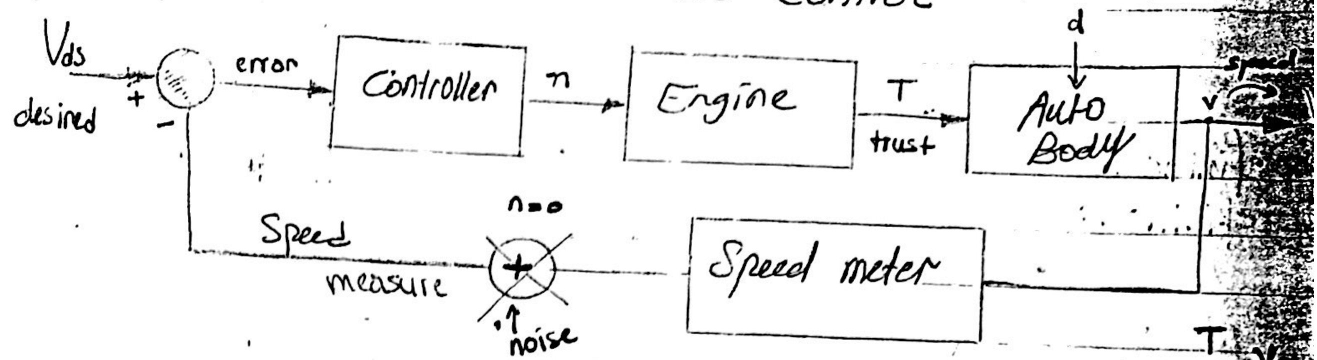
* dis advantages :—
- more complicated
- Output should be measu…
- Can go Unstable




F. R

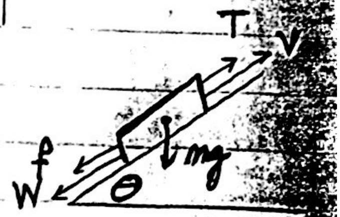
* Example (1) :-

## Cruise Control




Mathematical model :-

$$m\dot{V} = T - F - W$$




assume: $F = bV$ → friction.

$W = mg\sin\theta$ → force due to road grade. إنحدار

$T = au$ → throttle angle.

* at Steady State : $\dot{V} = 0$ $\left(\frac{d}{dt} = \dot{V}\right)$

but :- $m\dot{V} = au - bV - w$ → (1)

$m, a, b, w$ → are uncertain.

Open loop Control :- العلاقات غير معتمدة على قيمة (V)

to Calculate Control assuming : $w = 0$, $m = \hat{m}$, $b = \hat{b}$

nominal Values (Avg)

$\hat{m}V = au - \hat{b}V_{ds} = 0$ at Steady State

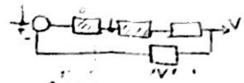
$u = \frac{\hat{b}}{a} V_{ds}$ → سرعة معينة

Analysis :- apply the Control (u) to the System and assume that $w \neq 0$ (constant), $b \neq \hat{b}$, $m \neq \hat{m}$

in (1) ⇒ $m\dot{V} = a\left(\frac{\hat{b}}{a}V_{ds}\right) - bV - w$

at Steady State $\dot{V} = 0$

$$V = \frac{\hat{b}}{b}V_{ds} - \frac{1}{b}W$$

* Closed Loop :- 

We Can design $U$ as : $U = K (V_{ds} - V) + \frac{\hat{b}}{a} V_{ds}$

High Gain feed back

feed forward

$m \neq \hat{m}$ , $b \neq \hat{b}$ , $w \neq 0$

in (1) $\rightarrow$ $m\dot{V} = ak(V_{ds} - V) - bV - W + \hat{b} V_{ds}$

at Steadystate :-

$$(ak + \hat{b}) V_{ds} - (b + ak) V - W = 0$$

$$V = \underbrace{\frac{ak + \hat{b}}{ak + b}}_{\simeq 1} V_{ds} - \underbrace{\frac{1}{b + ak}}_{\simeq 0} W$$

ask grows $\rightarrow$ V approach V desired.

---

5th March 2016.

* Laplace transform :-

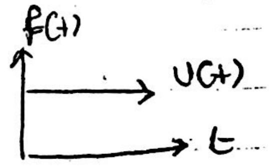
Mathematical method which can be used to convert differential equations into algebric equations in $S$ domain.

it's easier to get the Solution in ($S$).

Dif :- $\mathcal{L}\{f(t)\} = \int_0^\infty f(t) e^{-st} dt = F(S)$

$s = \sigma + j\omega$.

Example (1) :- find the Laplacian transformation (L.T) Step Unite function.




$$\int_0^\infty 1 \, e^{-st} dt = \left. \frac{e^{-st}}{-s} \right|_0^\infty = 0 - \left(\frac{1}{-s}\right) = \frac{1}{S}$$

* $\mathcal{L}(K U(t)) = \frac{K}{S}$

inverse.

$$\mathcal{L}^{-1}(F(t)) = \frac{1}{2\pi j} \int^{\sigma + j\infty} F(s) e^{St} dS$$

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* Example(2) :- find (L.T) for $f(t) = e^{at}$   $a \geq 0$

$$\int_0^\infty e^{at} e^{-st} dt = \int_0^\infty e^{-(s-a)t} dt = \frac{e^{-(s-a)t}}{-(s-a)} \Big|_0^\infty = \frac{1}{s-a}$$

Rules for laplace Transform :-

Sum of transformation

$$f_1(t) \pm f_2(t) \Leftrightarrow f_1(s) \pm f_2(s)$$

Multiplication by Constant (A)

$$A\,f(t) \longleftrightarrow A\,f(s)$$

Shifting in time, (T) delay :

$$f(t-T) \Leftrightarrow f(s)\, e^{-Ts}$$

first dirvitive

$$\frac{d}{dt} f(t) \longleftrightarrow s\, f(s) - f(0)$$

Second dirvitive

$$\frac{d^2}{dt^2} f(t) \longleftrightarrow s^2 f(s) - s\, f(0) - \frac{d}{dt} f(0)$$ n time

$n^{th}$ dirvitive

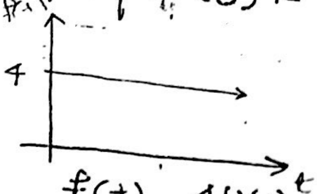
$$\frac{d^n}{dt^n} f(t) \longleftrightarrow s^n f(s) - \dots \frac{d^{n-1}}{dt^{n-1}} f(0)$$

Integration

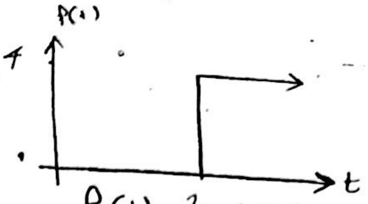
$$\mathcal{L}\left(\int f(t)\,dt\right) = \frac{f(s)}{s}$$

$$\mathcal{L}\left(\iint\dots\int f(t)\,dt^n\right) \Leftrightarrow \frac{f(s)}{s^n}$$

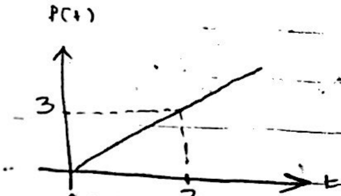
Example (3) :- find $F(S)$ :-



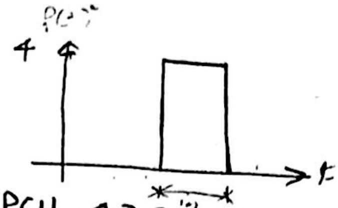
$f(t) = 4U(t)$
$F(S) = 4/S$



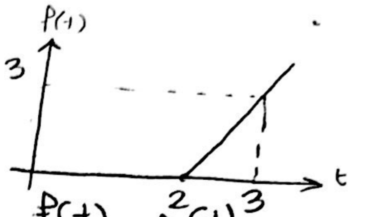
$f(t) = 4U(t-2)$
$F(S) = \frac{4}{S} e^{-2S}$



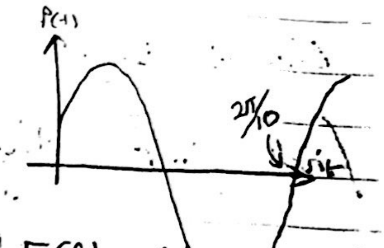
$f(t) = 3r(t)$
$F(S) = 3/S^2$



$f(t) = 4\delta(t-2)$
$F(S) = 4e^{-2S}$



$f(t) = r(t)$
$F(S) = \frac{3}{S^2} e^{-2S}$




$F(S) = \frac{\omega}{S^2+\omega^2} \to \omega =$
$F(S) = \frac{2\times 10}{S^2+100}$

Example (4) :- find $F(S)$ :-

$t^2 \to F(S) = 2!/S^3$

$t^2 e^{-at} \to F(S) = 2!/(S+a)^3$

$t^2 (1+e^{-at}) \to F(S) = \frac{2}{S^3} + \frac{2}{(S+a)^3}$ . #

Example (5) :- Solve Using Laplace transform.

$$\frac{3\,dx}{dt} + 2x = 4$$

$$3S\,X(S) + 2\,X(S) = \frac{4}{S} \to X(S) = \frac{4}{S(3S+2)}$$

from table : $A/S(S+A) \leftrightarrow 1 - e^{-at}$.

$$X(S) = \frac{4/3}{S(S+2/3)} = 2 \cdot \frac{2/3}{S(S+2/3)}$$

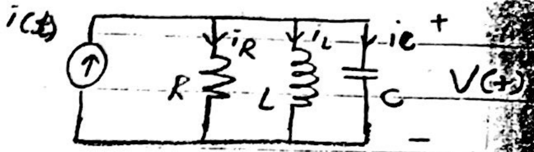
$$x(t) = 2(1 - e^{2/3 t}).$$

$U(t)$ : shows that the response is zero until $t=0$
Unless otherwise was specified = all inputs won't start
Until $t=0$.

# * Transfer function :- $G(s) = \frac{C(s)}{R(s)} = \frac{Pol(b)}{Pol(a)}$

It's the ratio of the Laplace transform of The Output to the laplace transform of the input (assuming zero initial condition).

Example (1) :-




find T.F $\frac{V(s)}{I(s)}$

## Method (1) :- Using differential equations.

$$i(t) = i_R + i_L + i_C$$

$$= \frac{V(t)}{R} + \frac{1}{L}\int_0^t V(t)\,dt + C\frac{dV(t)}{dt} + \cancel{i_L(0^-)}$$

$i_L(0^-) = 0$ → initial condition

$$\frac{di(t)}{dt} = \frac{1}{R}\frac{dV(t)}{dt} + \frac{1}{L}V(t) + C\frac{d^2V(t)}{dt^2}$$

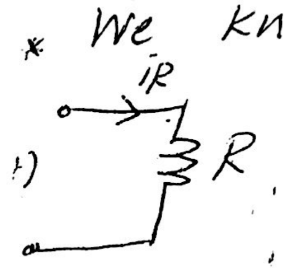
taking Laplacian transform for both Sides.

$$S\,I(s) = \frac{S}{R}V(s) + \frac{1}{L}V(s) + CS^2\,V(s)$$

$$\frac{V(s)}{I(s)} = \frac{S}{CS^2 + \frac{1}{R}S + \frac{1}{L}}$$

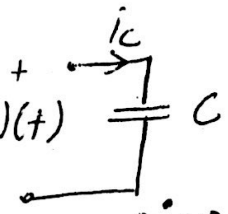
## Method (2) :-

* We know that :-



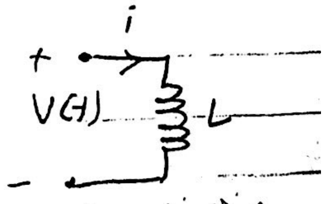

V = IR

V(s) = I(s)R




$$V(t) = \frac{1}{C}\int i(t)\,dt$$

$$V(s) = \frac{1}{CS} I(s)$$




$V(t) = L\,di/dt$

$V(s) = L\,S\,I(s)$

Impedance :-


$$Z(s) = \frac{V(s)}{I(s)}$$

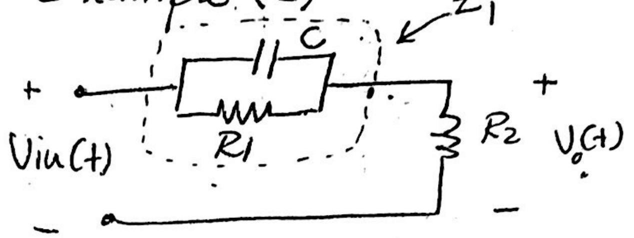
Resistance $\rightarrow R$
Capacitor $\rightarrow \frac{1}{CS}$
inductor $\rightarrow LS$

$$I(s) = I_R + I_L + I_C$$

$$I(s) = \frac{V(s)}{R} + \frac{V(s)}{LS} + CS\,V(s)$$

$$\frac{V(s)}{I(s)} = \frac{1}{1/R + 1/LS + CS}$$

Example (2) :-

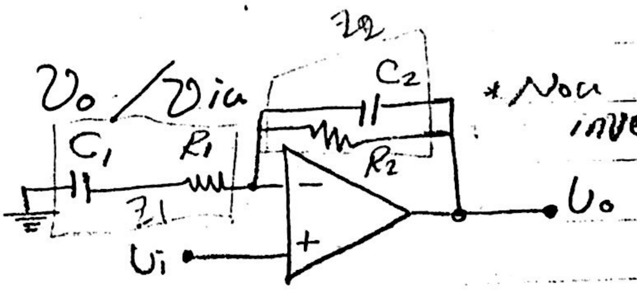



find T.f

$$\frac{V_o(S)}{V_{in}(S)} = \frac{R_2}{Z_1 + R_2}$$

$$Z_1 = \frac{R_1/CS}{R_1 + \frac{1}{CS}} = \frac{R_1}{R_1CS+1} \rightarrow \frac{V_o(s)}{V_{in}(S)} = \frac{R_2(SCR_1+}{R_1 + R_2(CSR_1+}$$

Example (3) :- find $V_o/V_{in}$




* Non inverting

$$\frac{U_o}{U_i} = \frac{Z_1 + Z_2}{Z_1} = 1 + \frac{Z_2}{Z_1}$$

$$\frac{U_o}{U_i} = 1 + \frac{R_f}{R_1} = 1 + \frac{Z}{Z_1}$$

$$Z_1 = R_1 + \frac{1}{C_1S} = \frac{R_1C_1S+1}{C_1S}$$

inverting :-

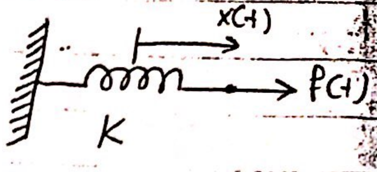
$$Z_2 = \frac{R_2 \times 1/C_2S}{R_2 + 1/C_2S} = \frac{R_2/\cancel{C_2S}}{R_2C_2S/\cancel{C_2S}} = \frac{\cancel{R_2}}{\cancel{R_2}C_2S} = \frac{1}{C_2S} \quad \frac{U_o}{U_i} = \frac{-R_f}{R_1}$$

$$\frac{U_o}{U_i}(S) = 1 + \frac{1}{\left(\frac{R_1C_1S+1}{C_1S}\right)C_2S} = 1 + \frac{1}{(R_1C_1S+1)\frac{C_2}{C_1}}.$$

# * Transfer function (mechanical systems) :-

## * Translational mechanic system :-

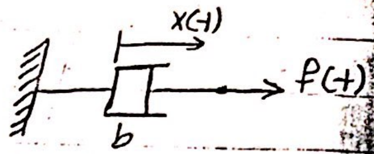
1). Spring :-




$$f(t) = K\,x(t).$$

$$F(S) = K\,x(S).$$

2). Viscous damper :-




$$f(t) = b\,\frac{dx(t)}{dt}.$$

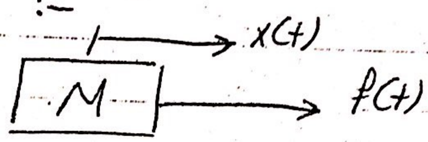
$$F(S) = b\,S\,X(S).$$

b ↳ damping constant

.. ≡ friction factor.

Also friction ...

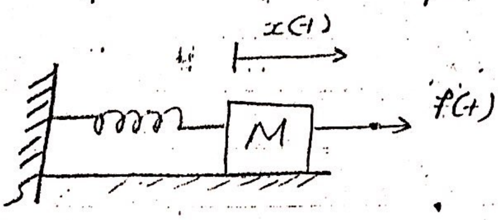
3). Mass :-




$$f(t) = M\,\frac{d^2X(t)}{dt^2}$$

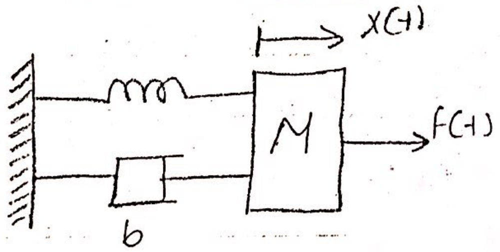
$$F(S) = M\,S^2X(S).$$

Example (1) :- find T.f $(X/F)$

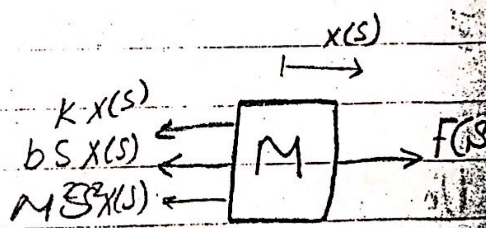



$$F = kX + bSX + MS^2X$$

$$\frac{X}{F} = \frac{1}{MS^2 + bS + K}$$




* free body Diagram :-



HW(1).

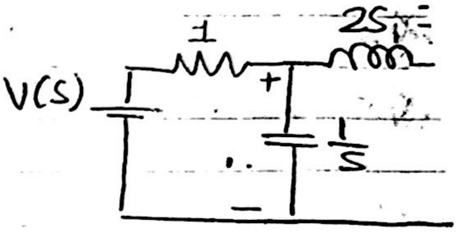
find $x(t)$ for the following :—

① $\frac{4S-2}{S^2-2S-8}$ ② $\frac{3S^2+10S-2}{S^2+3S+2}$ ③ $\frac{4(S+45)}{S(S^2+6S+45)}$

④ $\frac{S^2+4S+1}{(S+2)(S+1)^2}$

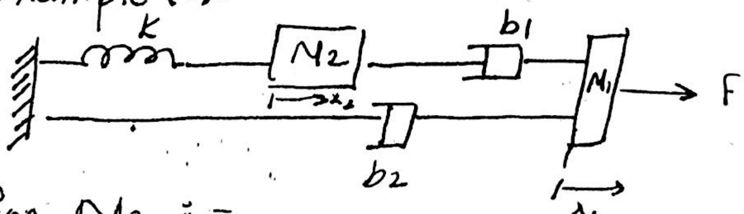
Use Matlab to find the partial fraction.

$$I(s) = \frac{V_1}{2S+1}$$

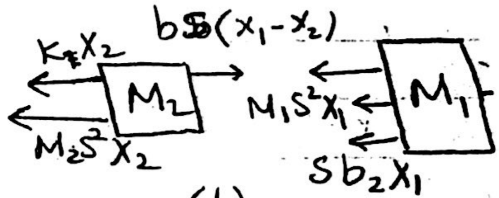



$$\frac{V_1}{V(S)} = (2S+1)/(2S^2+S+1)$$

Example (1):-




find $x_1/F$.




*for $M_2$ :-

$$KX_2 + M_2S^2X_2 - b_1S(X_1 - X_2) = 0$$

$$-bSX_1 + X_2(M_2S^2 + b_1S + K) = 0 \longrightarrow (1)$$

* for $M_1$ :-

$$M_1S^2x_1 + b_1S(x_1 - x_2) + Sb_2x_1 = F$$

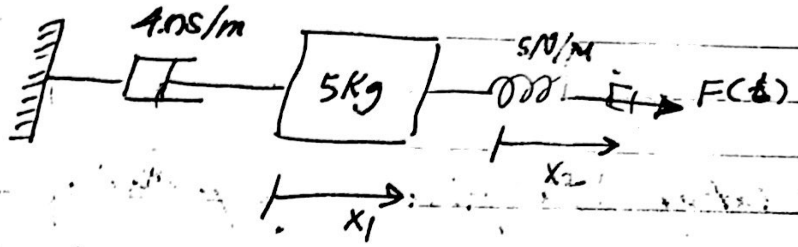
$$(M_1S^2 + (b_1+b_2)S)x_1 - b_1S\,x_2 = F \longrightarrow (2)$$

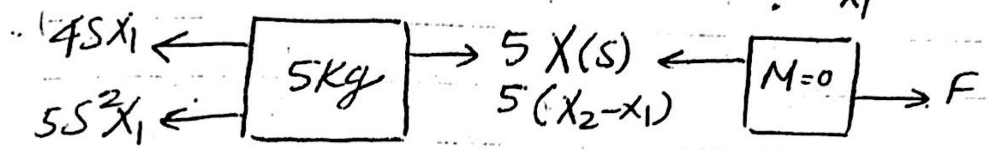
$$x_1 = \frac{\begin{vmatrix} F & -b_1S \\ 0 & M_2S^2+b_1S+K \end{vmatrix}}{\begin{vmatrix} M_1S^2(b_1+b_2)S & -b_1S \\ -b_1S & M_2S^2+b_1S+K \end{vmatrix}} = \frac{(M_2S^2+b_1S+K)F}{\Delta S}$$

$$\frac{X_1}{F} = \frac{M_2 S^2 + b_1 S + K}{\Delta S}$$

$$\Delta S = (M_1 S^2 + (b_1 + b_2) S)(M_1 S^2 + b_1 S + K) - (b_1 S)^2$$

find $\frac{x_1}{F}$




$$4Sx_1 + 5S^2X_1 - 5(X_2 - X_1) = 0$$

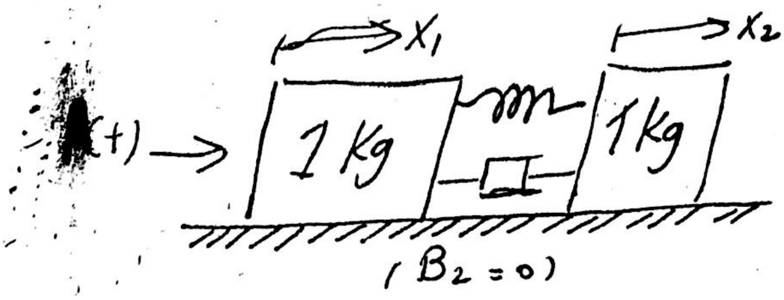
$$(5S^2 + 4S + 5)X_1 - 5X_2 = 0 \rightarrow (1)$$

$$5(X_2 - X_1) = F(S) \rightarrow (2)$$

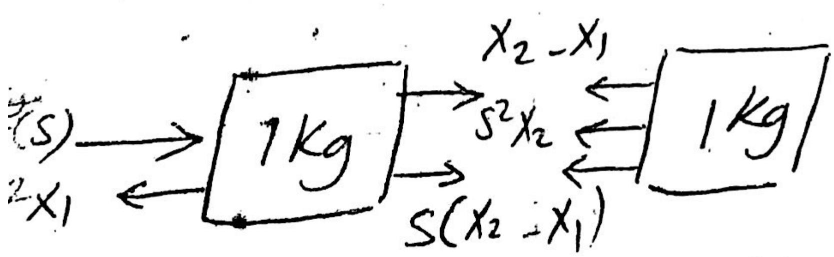
adding (1) to (2)

$$(5S^2 + 4S)x_1 = F(S)$$

$$\frac{X_1}{F} = \frac{1}{5S^2 + 4S} \ \#$$




find $\frac{x_2}{F}$



$$s^2 x_1 - (X_2 - X_1) - S(X_2 - X_1) = F$$

$$(S^2 + S + 1)X_1 - (S+1)X_2 = F \rightarrow (1)$$

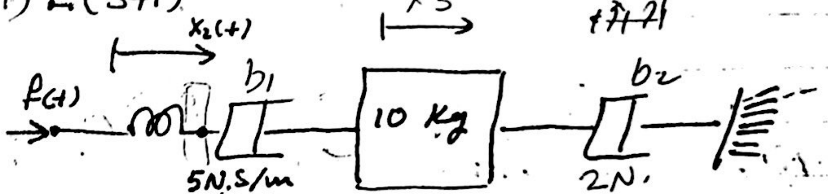
$$(X_2 - X_1) + S(X_2 - X_1) + S^2 X_2 = 0$$

$$-(S+1)X_1 + (S^2 + S + 1)X_2 = 0 \rightarrow (2)$$

$$X_2 = \frac{\begin{vmatrix} S^2+S+1 & F \\ -(S+1) & 0 \end{vmatrix}}{\begin{vmatrix} S^2+S+1 & -(S+1) \\ -(S+1) & (S^2+S+1) \end{vmatrix}} = \frac{F(S+1)}{(S^2+S+1)^2 - (S+1)^2}$$

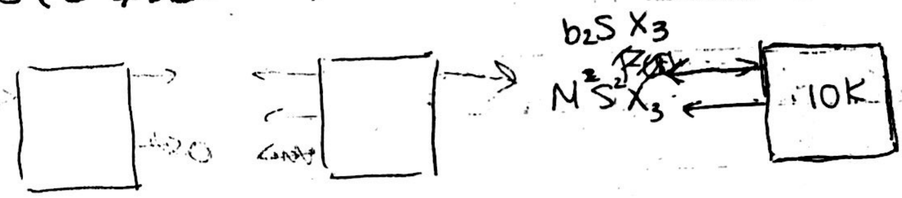
$$\frac{X_2}{F} = \frac{(S+1)}{(S^2+S+1)^2 - (S+1)^2}$$

# Exercise #




- Determine where to derate $x_1$ $x_1, x_3$
- find $x_1 / f$

result : $\left[ \frac{10}{S(S^2 + 50S + 2)} \right]$

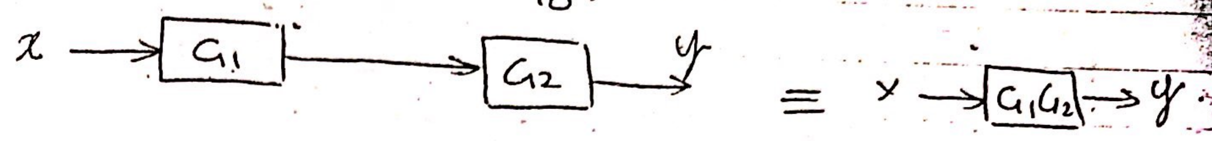


15th march 2016

## * Block Diagram Reduction :-

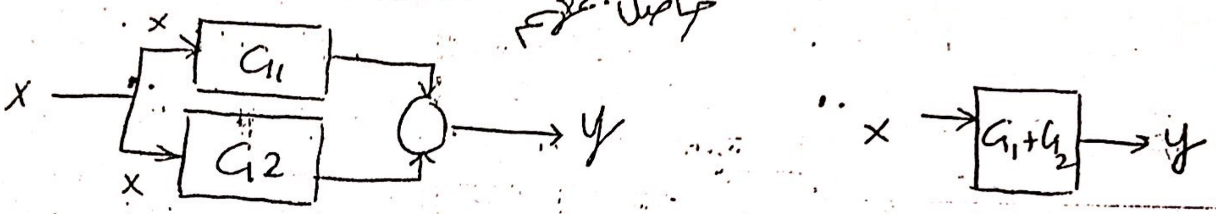
* Series Connection :

حاصل ضرب

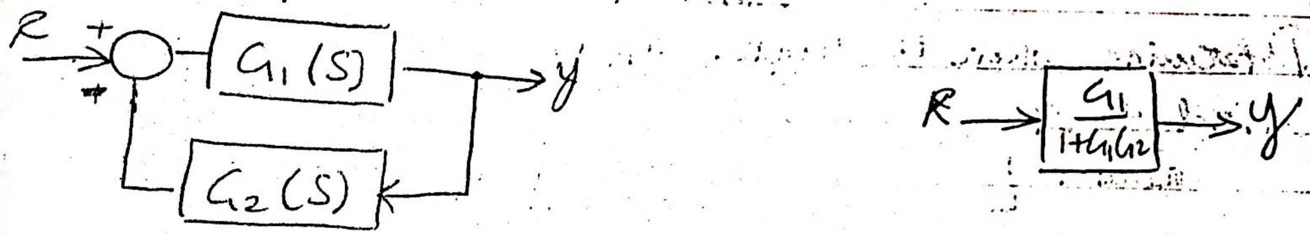



* Parallel Connection :-

حاصل جمع

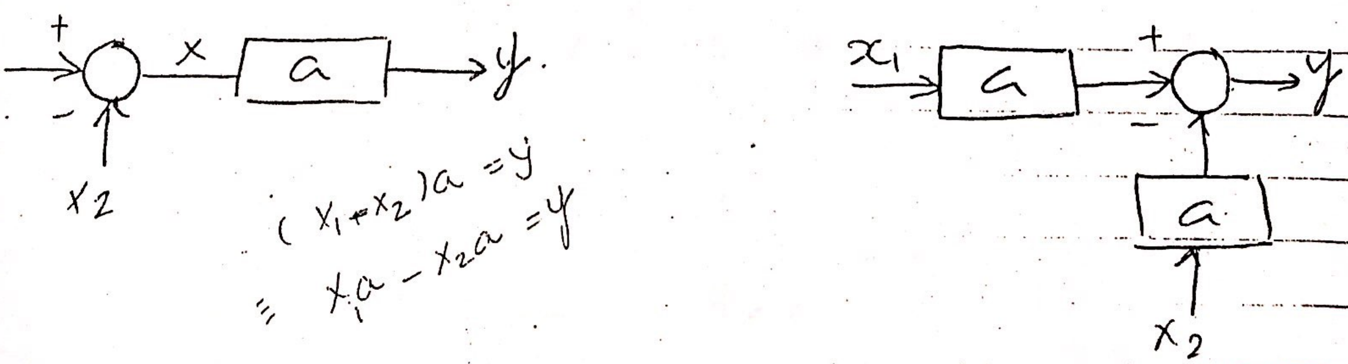



* Negative feedback :-




* Posetive feed back :-

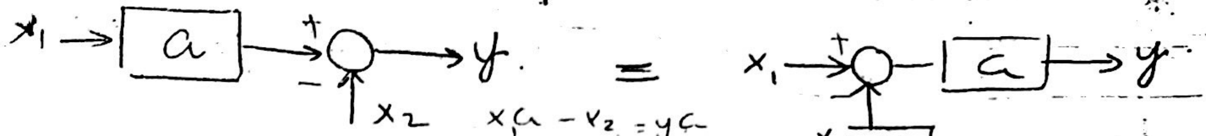

* Moving Block :-




$(X_1 + X_2)a = y$

$= X_1 a - X_2 a = y$

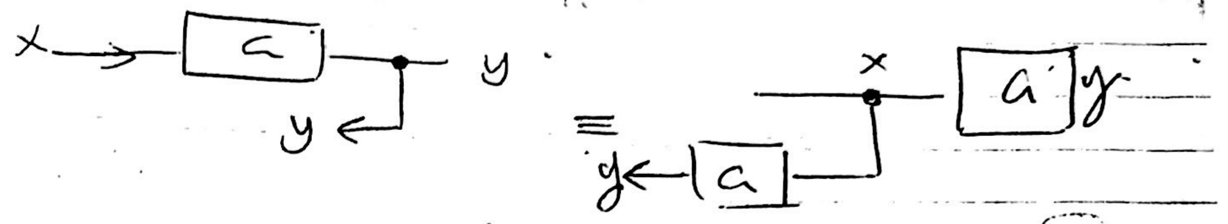
## * Moving of Summing Point :-

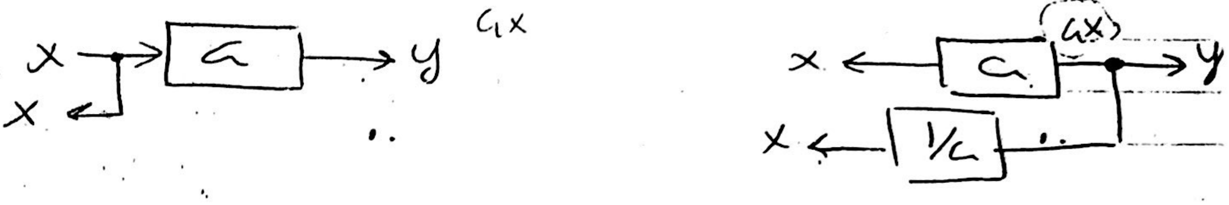


$x_1G - x_2 = yG$

$(x_1 - x_2)G = y$

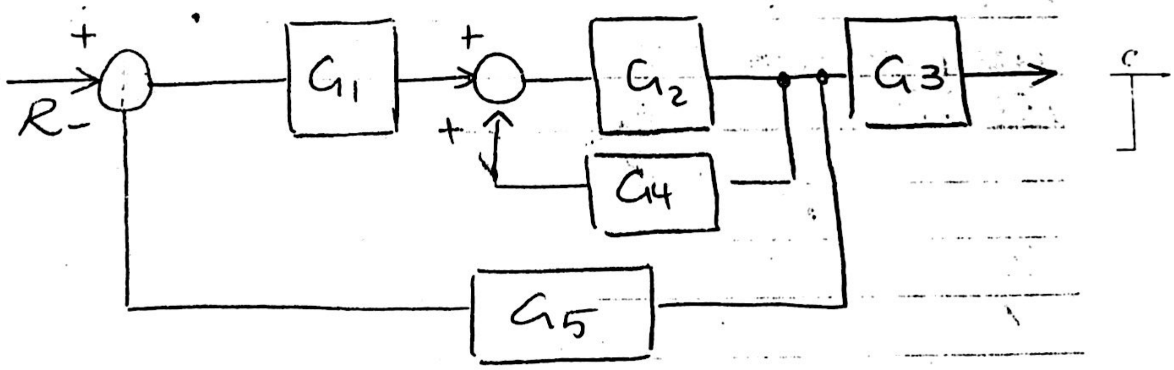
## Moving Branching :-

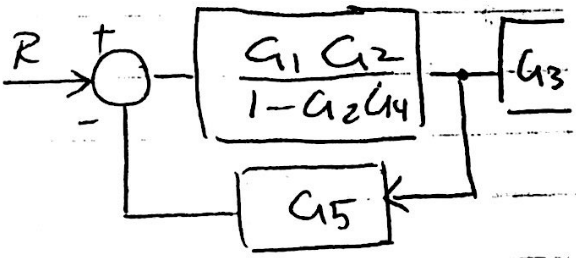


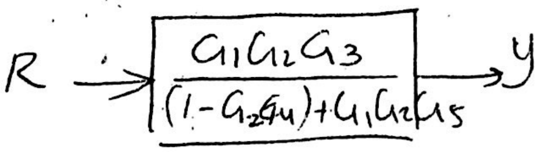


## * Example (1) :-

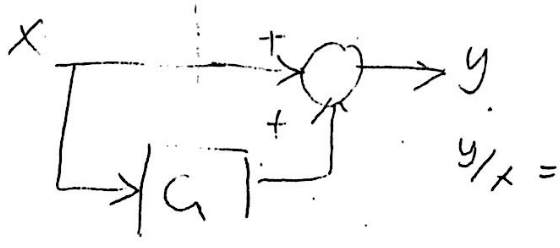
Find $G = y/R$







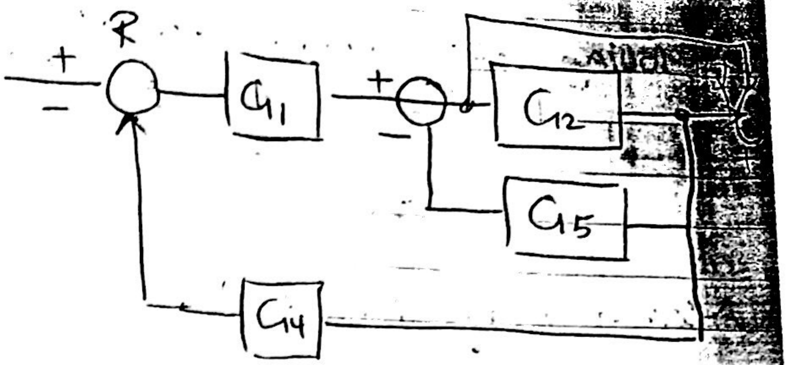
$$\frac{y}{R} = \frac{G_1G_2G_3}{(1-G_2G_4)+G_1G_2G_5}$$

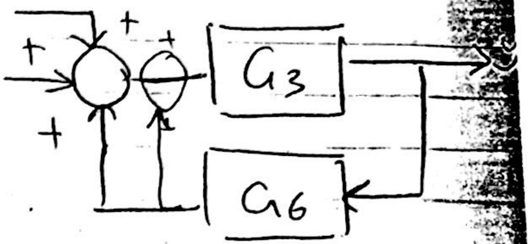


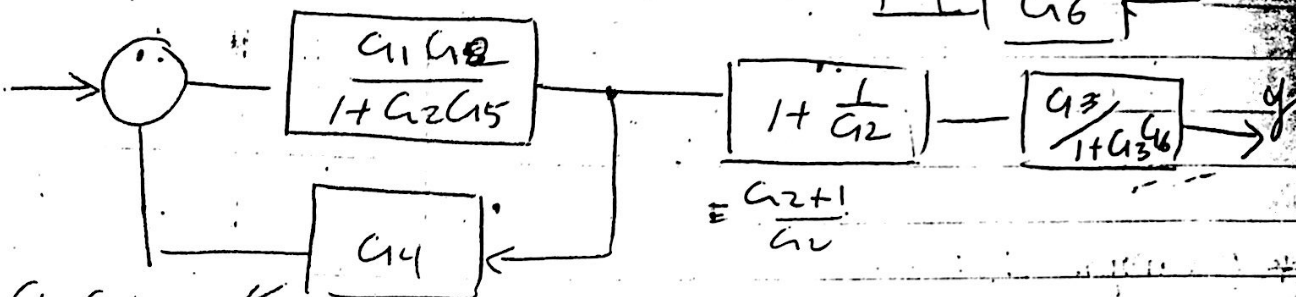
$y/x = 1 + G$

* Example (2):-

find $y/R$







$$\frac{\frac{G_2G_1}{1+G_2G_5}}{1+\frac{G_2G_1G_4}{1+G_2G_5}}$$

$$y/x = \frac{G_1G_3(G_2+1)}{(1+G_2G_5+G_4G_2G_1)(1+G_3G_6)}$$

5 [illegible]
5 homework
5 Quiz
15 Test I
10 test II
50. final

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# * Signal flow Graph

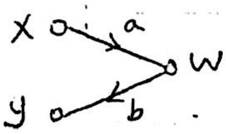
a signal flow graph consists of nodes directed branches.

$$x \longrightarrow \boxed{G(s)} \longrightarrow y \quad \equiv \quad x \circ \xrightarrow{\;G\;} \circ\, y$$

$$y = x\,G(s) \qquad\qquad y = Gx$$

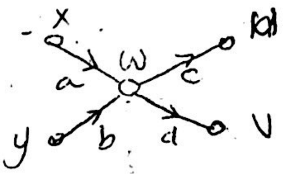
a node performs two functions :-

① - Addition of incoming signals.




$\therefore$ $w = ax + by$

② Transmission of the sum of incoming signals to outgo branches.



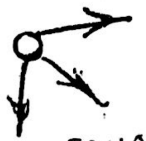

$w = ax + by$

$v = wc = (ax + by)c$

$u = wd = (ax + by)d$

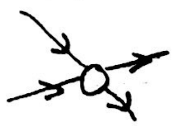
There are 3 types of nodes :-

2016 - 03 - 19



source node

sink node

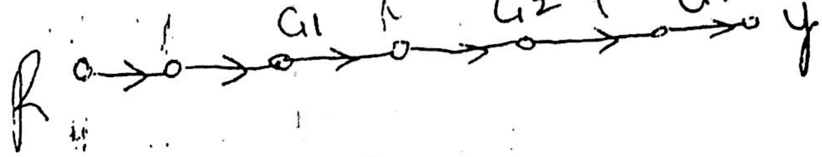


mixed nod

## * forward path :-

a branch or continuous sequence of branches from source node to a sink node such that a node appears only once on the branch

fig (a)



$P_k$ = $k^{th}$ forward path Gain :—

The product of branch gains to the $k^{th}$ forward path

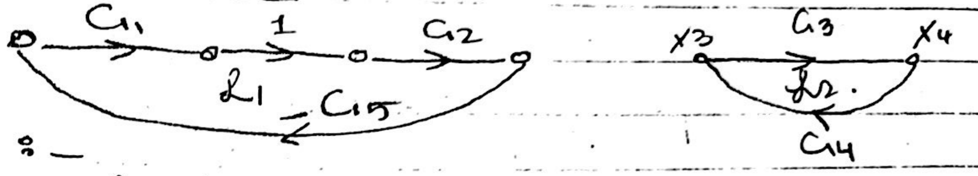
for fig (a)

$$P_1 = G_1 G_2 G_3$$

* loop :—

a Closed path that starts at a node and ends at the same node

[ Fig(b) ]




* loop Gain :—

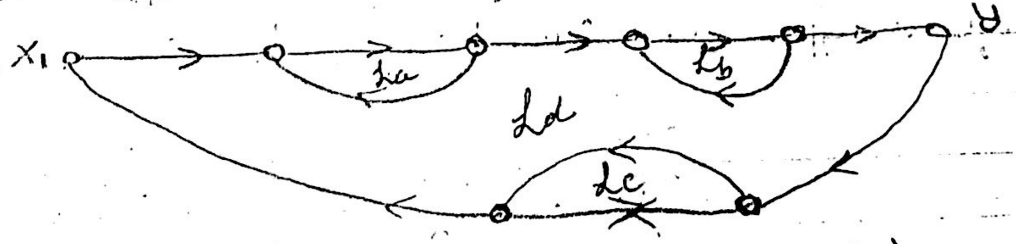
The product of branch gain in a loop.

$L_1 = G_1 G_2 G_5$ $L_2 = G_3 G_4$

[ Fig(b) ]:

* Non touching loop :— (a-b) (a-c) (b-c)

two loops are non-touching if they don't have a common node.




* Determinant of the Graph :— $(\Delta)$ :-

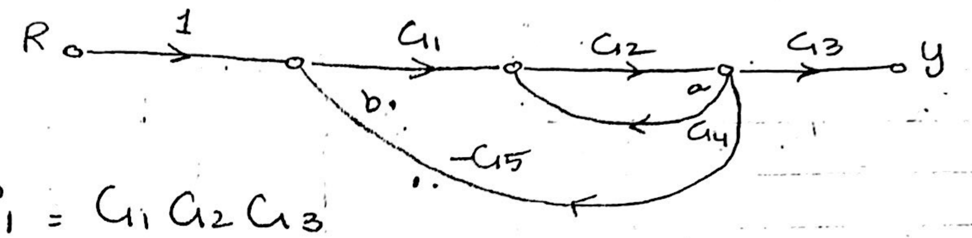
$\Delta$ = 1 — sum of loop gains (la + lb + lc) + sum of the product of gains of non-touching loops taken two at a time (la lb + lb lc + la lc) — sum of the product of gains (la lb lc) of non-touching loops taking 3 at a time + ....

$\Delta k$ = Determinant of the remaining graph after removing the loop that touches the $k^{th}$ forward path.

* Mason's Rule :— The transfer function from the input to the output is given by: $G = \frac{\Sigma \Delta k P_k}{\Delta}$

$$\frac{y}{R} = G = \frac{\Sigma_K P_K \Delta_K}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 \ldots}{\Delta}$$

* Example (1) :— find $y/R$ using Mason's Rule.



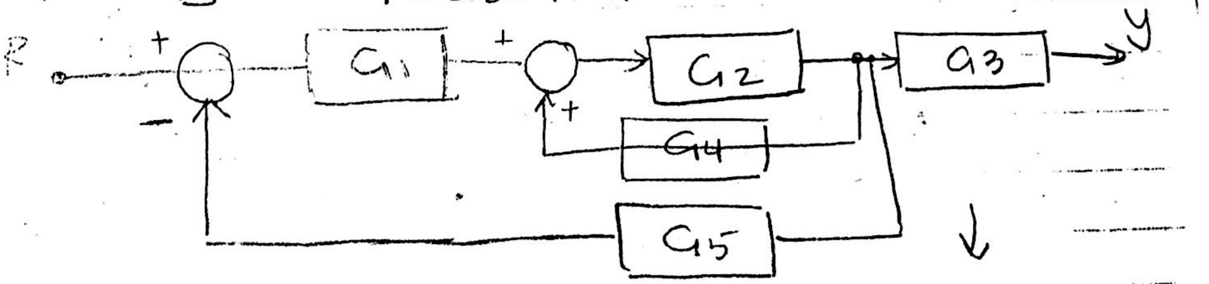

$P_1 = G_1 G_2 G_3$

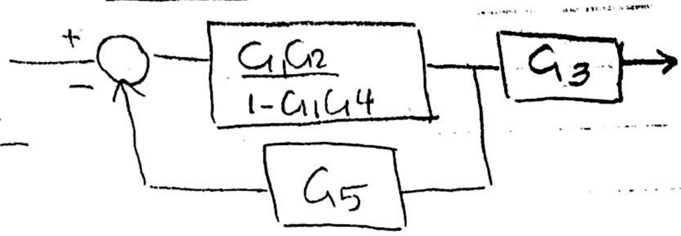
$L_a = G_2 G_4$. $\qquad L_b = -G_1 G_2 G_5$

$\Delta = 1 - (L_a + L_b) + \ldots$

$\Delta_1 = 1 - 0$

$$G = \frac{\Sigma P_k \Delta_k}{\Delta} = \frac{G_1 G_2 G_3 (1)}{1 - G_2 G_4 + G_1 G_2 G_5}$$




$$G_3 \times \frac{\frac{G_1 G_2}{1 - G_1 G_4}}{1 + G_5 \frac{G_1 G_2}{1 - G_1 G_4}}$$

$$G = \frac{y}{R} = \frac{G_1 G_3 G_2}{1 - G_2 G_4 + G_1 G_2 G_5}$$

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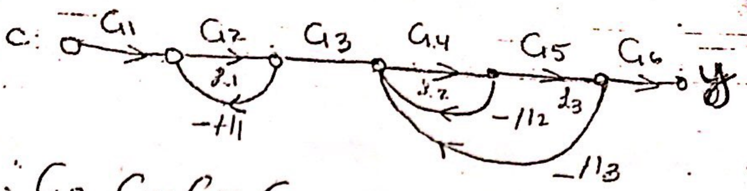
Example (2) :-




$P_1 = G_1 G_2 G_3 G_4 G_5 G_6$.

$L_1 = -G_2 H_1$.

$L_2 = -G_4 H_2$.

$L_3 = -G_4 G_5 H_3$.

* non touching :-

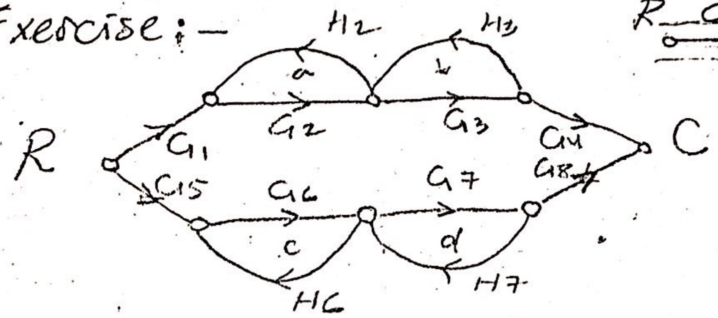
$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2 + L_1 L_3)$

$\Delta_1 = 1 - 0$.

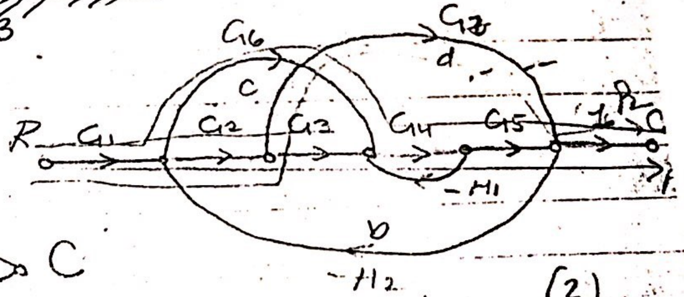
$$y/c = G = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2 G_3 G_4 G_5 G_6}{1 - (L_1 + L_2 + L_3)(L_1 L_2 + L_1 L_3)}$$

$$= \frac{\cancel{G_1 G_2 G_3 G_4 G_5 G_6}}{1 + G_2 H_1 + G_4 H_2 + G_4 G_5 H_3}$$

* Exercise :-




(1)




(2)

$P_1 = G_1 G_2 G_3 G_4 G_5$

$P_2 = G_1 G_2 G_7$

$P_3 = G_1 G_6 G_4 G_5$

$l_a = -H_1 G_4$

$L_b = -H_2 G_2 G_3 G_4 G_5$

$L_c = -H_2 G_6 G_4 G_5$

$L_d = -H_2 G_2 G_7$

$\Delta = 1 - (L_a + L_b + L_c + L_d) + (L_a L_d)$

$\Delta_1 = 1 - 0$

$\Delta_2 = 1 - l_a$

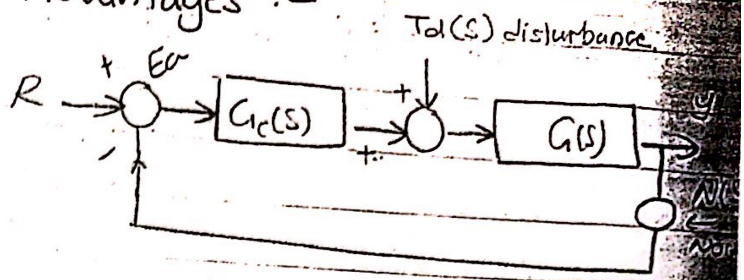
$\Delta_3 = 1 - 0$

$G =$

## * feed back System Advantages :-

①. Sensetivity.

②. Tracking error.




$U$ = Control Signal

$E$ = tracking error. $= R - Y \longrightarrow (*)$

$$E = R - y.$$

$$E_a = R - (y + N(s))$$

$$Y = (Td(S) + U)G(s)$$

$$= (Td(S) + Gc(S)E_a)G(s)$$

$$= [Td(S) + Gc(R - (y + N(S)))]\,G(S)$$

$$Y(1 + GcG) = GGcR + G\,Td - GGcN$$

$$Y = \frac{GGc}{1+GGc}R + \frac{G}{1+GGc}Td - \frac{GGc}{1+GGc}N.$$

$$\downarrow$$

$$(*)\quad E = R - y = R - (\qquad)$$

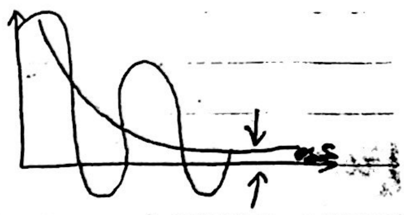
$$= \left(1 - \frac{GGc}{1+GGc}\right)R - \frac{G}{1+GGc}Td + \frac{GGc}{1+GGc}N.$$

* By increasing $Gc(s)$, we reduce the error due to R, and Td

for the Noise we have to Use transfer function to filter the noise.

* Steady State Tracking Error:-

$$e_{ss} = e(t = \infty)$$




* Final Value Theorem:-

$$e(\infty) = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

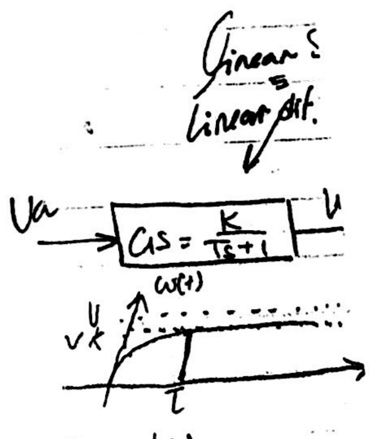
for the pervious figure let, $N = 0$, $T_d = 0$
let $r(t)$ is the unite step $(u(t))$.

$$R(s) = \frac{1}{s}$$

$$E(s) = \frac{1}{1 + G_c(s)G(s)} R(s)$$

$$e_{ss} = \lim_{s \to 0} s\,E(s) = \lim_{s \to 0} \frac{\cancel{s}}{1 + G_c(s)G(s)} \cdot \frac{1}{\cancel{s}}$$

$$= \frac{1}{1 + G_c(0)G(0)}$$




*③ Sharping the transient response:

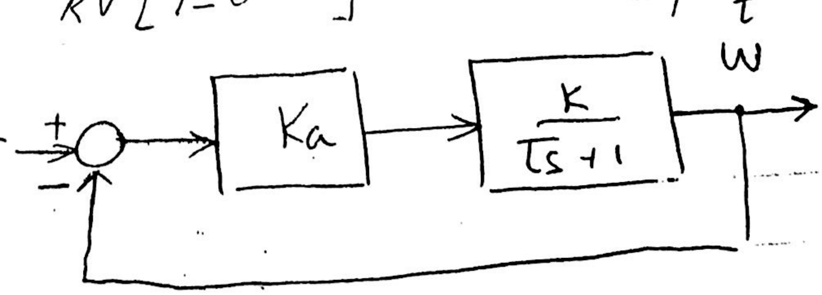
$\dashrightarrow V/s$

$$V_a(t) = V u(t)$$

$$W(s) = \frac{KV}{(\tau s+1)s}$$

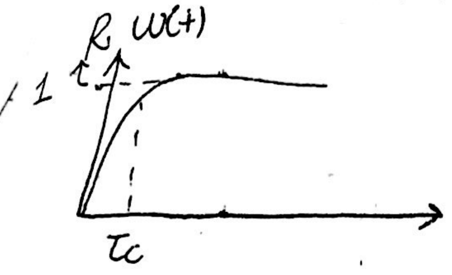
$$W(t) = KV\left[1 - e^{-t/\tau}\right]$$

* for: C.I.S :-



$$\therefore T = \frac{W}{R} = \frac{KK_a/\tau s+1}{1+(\quad)} = \frac{K K_a}{\tau s + 1 + K K_a}$$

$$= \frac{KK_a/1+KK_a}{\frac{\tau}{1+KK_a}s + 1} = \frac{K_c}{\tau_c S + 1}$$




By increasing $K_a$ we reduce $\tau_c$ that means faster response which is better.